PG – 144

III Semester M.Sc. Degree Examination, December 2014
(Semester Scheme) (NS)
MATHEMATICS
M 303 : Differential Geometry

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer* ***any five*** *questions choosing atleast* ***two*** *from* ***each*** *Part.*
ii) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Define : (i) natural coordinate functions (ii) vector field (iii) natural frame field to E^3. Prove that every vector field is a linear combination of natural frame field. **6**

b) Let f and g be functions on E^3, v_p be a tangent vector to E^3, a and b be real numbers. Then prove that

i) $v_p[af + bg] = a\, v_p[f] + b\, v_p[g]$,

ii) $v_p[fg] = v_p[f]g(p) + f(p)v_p[g]$. **6**

c) Prove the identity $V = \sum_{i=1}^{3} V[x_i]U_i$, where V is a vector field and x_1, x_2, x_3 are natural coordinate functions. **4**

2. a) Let α be a curve in E^3 and let f be a differentiable function on E^3. Then prove that $\alpha^1(t)[f] = \frac{d}{dt}(f(\alpha))(t)$. **5**

b) Evaluate 1-form $\phi = x^2dx - y^2dz$ on the vectorfield $\frac{1}{x}V + \frac{1}{y}W$, where $V = xU_1 + yU_2 + zU_3$ and $W = x^2yU_1 + y^2zU_2 + z^2xU_3$. **6**

c) Find F_* for the mapping $F = (x \cos y,\ x \sin y,\ z)$ and compute $F_*(v_p)$ if $v = (2, -1, 3)$ and $p = (0, 0, 0)$. **5**

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3. a) Compute the Frenet apparatus k, τ, T, N, B of the unit speed curve $\beta(s) = \left(\frac{4}{5}\cos s, 1-\sin s, -\frac{3}{5}\cos s\right)$ show that this curve is a circle. Find its centre and radius. **7**

b) Prove that a regular curve α with k > 0 is a cylindrical helix if and only if τ/k is constant. **5**

c) Let $V = -yU_1 + xU_3$ and $W = \cos x U_1 + \sin x U_2$. Express $\nabla_V (Z^2 W)$ in terms of U_1, U_2, U_3. **4**

4. a) Define connection forms. Let w_{ij} $(1 \le i, j \le 3)$ be the connection forms of a frame field E_1, E_2, E_3 on E^3. Then for any vector field V on E^3 prove that $\nabla_V E_i = \sum_j w_{ij}(V) E_j$ $(1 \le i \le 3)$. **6**

b) With usual notations prove that $d\theta_i = \sum_j w_{ij} \wedge \theta_j$ $(1 \le i \le 3)$. **6**

c) Define a translation on E^3. If S and T are translations on E^3, prove that ST = TS is also a translation. **4**

PART – B

5. a) Define : (i) coordinate patch (ii) proper patch. If f is a differentiable real valued function on a non-empty set D of E^2, then show that $x : D \to E^3$ defined by x(u, v) = (u, v, f(u, v)) is a proper patch in E^3. **6**

b) Prove that a plane is E^3 is a simple surface. **4**

c) Let g be a differentiable real valued function on E^3 and c a number. Then prove that the subset M : g(x, y, z) = c of E^3 is a surface if and only if the differential dg is not zero at any point of M. **6**

6. a) Explain parametrization of surface of revolution. Find the parametrization of the surface obtained by revolving $C : (z-3)^2 + y^2 = 1$ around y – axis. **4**

b) Let $F : M \to N$ be a mapping of surfaces and let ξ and η be p-forms (p = 0, 1, 2) on N. If F^* is the pull back function, then prove that $F^*(\xi \wedge \eta) = F^*\xi \wedge F^*\eta$. **4**

c) Show that a mapping $X : D \to E^3$ is regular if and only if the u, v-partial velocities $X_u(d), X_v(d)$ are linearly independent for all $d \in D$, $D \subset E^2$. **8**

7. a) Obtain the shape operator of the saddle surface $M : z = xy$. **6**

   b) Define a umbilic point. Show that every point on a sphere is umbilic. **5**

   c) Let k_1, k_2 and e_1, e_2 be the principal curvatures and vectors of M at P, where $M \subset E^3$. Then prove that $k(u) = k_1 \cos^2\theta + k_2 \sin^2\theta$, where $u(\theta) = \cos\theta e_1 + \sin\theta e_2$. **5**

8. a) With usual notations prove that $K = \dfrac{ln - m^2}{EG - F^2}$ and $H = \dfrac{Gl + En - 2Fm}{2(EG - F^2)}$. **6**

   b) Compute K and H and hence k_1, k_2 for the surface of helicoid given by $X(u, v) = (u \cos v,\ u \sin v,\ bv)$; $b \neq 0$. **6**

   c) Define a geodesic curve. Prove that a geodesic on a plane is a straight line. **4**

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